NASA TECHNICAL NOTE

NASA TN D-4285

c. 1

NASA TN D-4285

FACTORS AFFECTING THE DESIGN AND USE OF A PHOTOGRAPHIC SEXTANT FOR SPACE NAVIGATION

by Thomas M. Walsh

Ames Research Center
Moffett Field, Calif.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • DECEMBER 1967

NASA TN D-4285

FACTORS AFFECTING THE DESIGN AND USE OF A PHOTOGRAPHIC SEXTANT FOR SPACE NAVIGATION

By Thomas M. Walsh

Ames Research Center
Moffett Field, Calif.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – CFSTI price $3.00

FACTORS AFFECTING THE DESIGN AND USE OF A PHOTOGRAPHIC SEXTANT FOR SPACE NAVIGATION

By Thomas M. Walsh

Ames Research Center

SUMMARY

A study was made of design and use factors that are pertinent to the fabrication of a photographic instrument for obtaining sextant-type data and for providing target star identification information. The objective was to adapt the highly accurate techniques of astronomical photography and film readout to the space cabin environment.

Of all the factors studied, the film was the most critical. Currently available emulsions lack either resolution, speed, or ease of development. Improvements in this area are expected and in the near future there should be no serious obstacles to sextant feasibility. However, current emulsions, if made available on flat glass plates, will suffice for sextant measurements that do not require landmark targets. Five basic sextant configurations are examined and one hardware concept is presented. A photographic sextant should be considered a serious alternative as a navigation instrument with a probable precision of better than 10 seconds of arc, with the added capability of providing data for target star identification.

INTRODUCTION

The marine sextant and the bubble sextant have long been tools for obtaining the measurements required in the computation of a line of position on the Earth's surface for ships and airplanes. The navigator measures the angle above the horizon of a known star or the Sun's limb. Errors of about 20 seconds of arc are compatible with the precision required in the usual nautical or aircraft navigation problem.

Space navigation requires an instrument for measuring the angle from a planetary body to a stellar reference or for measuring a planet's subtense for range information. One such instrument is a space sextant. Many configurations capable of making a star-to-planet measurement are conceivable; many types have been proposed; and several prototype models have been built. The Gemini Earth-orbital series of flights has inaugurated space testing of two models. A photographic sextant that can make conventional sextant measurements and simultaneously provide positive means for target star identification is investigated in this report.

The need for measuring an angle precisely in space navigation is brought out in the literature. For example, reference 1 shows that an increase in sextant error from 10 to 50 seconds of arc causes a highly undesirable increase in range and velocity deviation at perigee during a lunar roundtrip mission. References 2 and 3 analyzed 3 to 5 velocity corrections based upon 426 sextant-type measurements and showed that bias errors of 10 seconds of arc were too great to warrant the use of long sighting schedules. Therefore, it appears that a sextant for precise space navigation should yield an error of less than 10 seconds of arc.

The photographic sextant investigated would be used to photograph two or more stars and a planetary body simultaneously. The photographs would be used first to make measurements required for computing the angles between the stars and the planet and, second, to positively identify the target stars. The sextant would accumulate raw data in the form of photographs in a short time at low cost in terms of vehicle stabilization and electrical power. To be considered practical for midcourse navigation, the precision of the photographic sextant must be better than 10 seconds of arc and the sextant must have a sufficiently large field of view for positive identification of target stars. Furthermore, for space cabin conditions similar to those of Gemini and Apollo the size of the sextant must be minimized.

To determine the factors pertinent to the design, use, and fabrication of a photographic sextant, 17 factors were studied. Schematic layouts of sextant types were made to examine how factors of use and hardward requirements are affected by layout. Prototype sextants were built to obtain film and target data.

NOTATION

℄	center line of field of view of line of sight perpendicular to R line
EFL	equivalent focal length
f/number	focal length for target at infinity divided by diameter of aperture
LOS	line of sight
n_1	index of refraction in space, 1.000000
n_2	index of refraction of glass, 1.458
n_3	index of refraction of cabin atmosphere, 1.000094 (70° F, 5.1 psi)

R line	great circle through optical axes of both lines of sight, on the plane of the angle α_M
α	angle between the two targets
α_M	angle between the two lines of sight
δ	angle between normal to window and bisector of sextant angle α_M
ϕ	$\alpha - \alpha_M$; change in α due to refraction of cabin atmosphere, assuming no change due to the window

SEXTANT OPERATIONAL REQUIREMENTS

A navigator operating a photographic sextant on board an Apollo-type spacecraft must acquire the targets through a window approximately 12 inches in diameter. The maximum angular separation of the targets is assumed to be 60°. The operator will use the sextant to acquire the targets in the fields of view of their respective optical systems by pointing the lines of sight and adjusting the sextant angle. Both fields of view will be photographed simultaneously, the time will be recorded, and the angle between the lines of sight of the two fields of view will be noted via a suitable sextant angle pickoff. The spacecraft will be in a loosely controlled mode of oscillation so that the celestial scene will move past the window. One operational requirement will be that the bisector of the sextant angle be approximately normal to the window. A second requirement will be a spacecraft attitude control limit cycle of not more than $\pm 5^{\circ}$ to keep the planet and star targets within view from the window. Fiducial marks on each photograph will indicate the location of the optical axis of its field of view and also the plane of the sextant angle. The unique ability of the photographic sextant to entrap the necessary target data by a fast exposure despite vehicle motion reduces the requirement for the tighter vehicle stabilization needed by other sextants.

The film will be developed immediately after exposure, and must be on a stable base and be quickly dried so that subsequent handling and measurement operations can proceed. The target stars on the film must be visible to the unaided eye. They will be identified by their orientation within a known constellation, and will be confirmed by direct comparison of the photographs with reference star charts of the constellation to about the same scale.

The distance from the target image to the optical axis fiducial marks will be measured with a microscope. Figure 1 indicates, on the planet and star photographs, the four measurements (X_M, Y_M, X_S, and Y_S) to be made. Figures 2 and 3 indicate the spherical geometry, the four measurements of figure 1, and the angle pickoff reading α_M, which are all necessary to compute the angle between targets. The equations for the true sextant angle, shown for the two possible target positions, have been derived in the appendix. The equations for determining vehicle position from sextant measurements are presented in reference 4. The spacecraft computer could be used for these computations, with the sextant measurements as inputs to a computing program.

Confirmation of the film measurements, as well as identification of the target star and its coordinates, can be checked at any time by the same operator or by a second astronaut to increase the reliability of the input data.

The sextant can be calibrated in its environment with navigation stars as targets. The instrument zero bias can be determined by photographing the same star in both fields of view. With two known stars in the same field of view, the scale factor of the film can be determined since the star separation can be computed from stored data of right ascension and declination.

FACTORS OF SEXTANT DESIGN AND USE

To determine whether the photographic space sextant concepts discussed above can be realized with existing technology, it is necessary to examine the basic elements involved. Seventeen specific areas that influence the design of an actual instrument were investigated to determine the adequacy and feasibility of known techniques to accomplish the task. To achieve an overall accuracy of better than 10 seconds of arc, a goal of about 1 second of arc accuracy was established for each sextant parameter bearing on precision. The results of these studies and the design criteria to which they led are discussed below.

Image Size

The image size of the dimmest star to be captured on film should be easily discernible to the unaided eye at a normal viewing distance of about 10 inches. In making an identification, the navigator will compare the star photograph and the star charts, so, for convenience, the format size and image size should be such that the photograph can be studied readily and handled easily within the constraints of the capsule environment. Thus, no burdensome imaging or magnifying system should be required for identification. A 3-1/2- by 4-1/2-inch format obtainable from nominal 4- by 5-inch film packs would appear suitable.

Images of various magnitudes and sizes were obtained on Polaroid 4- by 5-inch prints to get an insight into the problems involved in detecting star images on film. Star image diameters of 0.02 inch were found to be satisfactory. A density change of the star image from a film background greater than 0.1 was detectable. The ambient light level used for observing the prints corresponded to a luminance of 17 ft-L for a magnesium-oxide surface with a reflectance near 1.0. The films used, the fog level, and the star background were such that the background on the prints had a luminance of about 1 ft-L. Under these conditions, a 0.02-inch star image subtends 7 minutes of arc at a viewing distance of 10 inches and has a contrast ratio of 0.26. Reference 5 indicates a 50-percent probability of detecting a uniform circular target of 7-minute subtense against a background luminance of 1 ft-L with a contrast ratio of only 0.0117, under the conditions of the Tiffany experiments. A 0.99 probability of detection would result from a contrast ratio 2.2 times this, or 0.026. In the Tiffany experiments (ref. 5), the observers felt confident of having "seen" a stimulus when the level of probability of detection was greater than 0.90. Detection of the sextant star image on a photographic plate in the spacecraft environment for ambient light conditions does not bear a one-to-one relationship to the Tiffany experiments, but there are many common elements. The sextant target minimal contrast of 0.26 is an order of magnitude greater than that required for the 0.99 confidence target of the Tiffany experiments. For the conditions of this study, the star photographs were readily detectable, and with the aid of the Tiffany data the probability of detection for other space cabin light conditions could be estimated. For all further computations, a star image of 0.02 inch is adopted as the desired size for the dimmest star to be captured.

Package Size and Weight

Size considerations influence the feasibility of the photographic sextant. It must be hand-held, pointed at targets, manually operated, and used behind a window of an Apollo-type vehicle in such a manner that many measurements may be made and the navigation problem pursued without time loss due to equipment shifting or navigator shifting to accommodate an unusually bulky instrument in the restricted volume environment of such a spacecraft. The weight of the sextant will not be felt in the space environment. Its inertia, however, will affect the pointing ease of the sextant. Work with gimbaled sextants with equivalent weight distributions indicates that there is no problem in this area. However, since weight affects the boost power requirements, it should be minimized. In line with these considerations, several optical configurations were developed with a periscope to keep the bulk of the sextant near the window. A small amount of folding of the optical path is also permitted by the periscope design to help reduce the overall size. It was concluded that a maximum package size of 20 by 16 by 11 inches would be reasonable.

Focal Length

Focal length is the major parameter that determines the precision of the instrument. The longest focal length will yield the most favorable ratio of seconds of arc subtense per inch of photograph. Because of the large format size, Cassegrain-type optical systems must be excluded since the opening in the primary mirror would be larger than the 3-1/2- by 4-1/2-inch format, necessitating a primary mirror diameter too large for the maximum package size. Furthermore, serious vignetting problems at the edge of the field would be encountered with such a large opening in the primary mirror, and would be detrimental to the identification task. This aspect of vignetting is discussed under Shutter. To gain long equivalent focal length (EFL) in a short distance, a positive lens followed by a negative lens, as in a Barlow configuration, may be used. Figure 4 indicates the combinations of focal lengths of positive and negative lenses that will achieve a 20.6-inch and a 24-inch EFL in a 16-inch net length. The 16-inch net length is fixed by the maximum package size previously assumed. The use of the 24-inch focal length combination leads to lower f/number requirements, and hence a higher degree of correction is necessary in the lens design to achieve a good lens. However, the added sensitivity of this longer focal length is worth the penalty, and a 24-inch EFL was adopted for subsequent analysis.

From the plots for a 24-inch EFL optical system, the largest f/number for the negative lens occurs at a focal length of 8 inches for this negative lens. The positive lens focal length would be 12 inches for a 4-inch aperture. The table in figure 4 shows the f/numbers to be 3 and -2 for the positive and negative lenses.

Aperture

Aperture, time of exposure, speed of film, vignetting, flare, etc., bear on the success of obtaining a given magnitude star image on film. Of these factors, aperture is the most important design parameter of the sextant pertaining to acquisition of a star image; it is limited by the practical limit of f/number for the elements in the 24-inch EFL system. A 4-, 5-, or 6-inch aperture would be reasonable from a size and weight standpoint. However, the 5- and 6-inch apertures would lead to very low f/numbers of 1.6 and 1.3, respectively, for the negative lens. The 4-inch aperture results in a more reasonable f/2 negative lens; therefore, this aperture was selected for subsequent calculations.

Scale Factor

The scale factor of the film depends on the choice of focal length and dictates the precision requirements of the film readout task. The 24-inch EFL leads directly to a scale factor of 1.16×10^{-4} inch/sec of arc in the center of the image plane. Without additional lenses the star images would focus on a curved (Petzval) surface, so a field-flattening lens would be required to image the stars on a flat photographic plate. Distortions due to this lens and to the Barlow lens must be removed from the sextant angle computations. All linear measurements on the flat photographic plate are made to the optical axis. A linear scale factor of 1.16×10^{-4} inch can be applied to these measurements and corrected when the angle is computed by the computer. The correction would have to be determined by calibrating the complete lens system for scale factor deviations from linearity at the photographic plate. Using as high an f/number as possible for the lens elements will keep the correction to a minimum. It will be assumed that a scale factor correction dependent upon the aberrations of the optical system will be applied to the basic scale factor when the sextant angle is computed.

Fiducial Lines

To facilitate readout of the photograph, fiducial lines are required on the film to indicate the plane of the angle between the sextant lines of sight. The location of the optical axis of each line of sight must also be indicated. Both can be accomplished by projecting fiducial patterns onto the film plane. A pattern containing the necessary information is the paired line shown in figure 1. The optical axis near the film center is determined by the fiducial pattern. The plane of measurement is located by the horizontal (long dimension of film) fiducial marks. The space between the paired lines (fig. 1) should be three times the microscope crosshair width, as can be deduced from reference 6. The readout microscope can then be used to measure the distance from a target to the optical axis, one measurement being parallel to the plane of the sextant angle and one perpendicular to this plane.

The light source and projector for the fiducial pattern could be mounted on the optical axis in front of the objective lens and should be small, say less than 3/4 inch in diameter, so as to obscure as little of the aperture as possible. The camera optics would transfer the fiducial pattern to the image plane at the time of exposure.

Readout Microscope

Microscope magnifications of 30 and 100 were used to measure the position of the image on the film. It was soon apparent that the reticle line thickness relative to star diameter was the most significant parameter that influenced the precision of the measurements. A change in magnification by a change in the microscope objective lens changed the size of the star image but not the reticle size. Since 30 power yielded a larger field of view than the 100 power, it was used to make star acquisition easier. Various ratios of star-to-reticle width were then obtained by changing the reticle line width. Powers lower than 30 were ruled out because of the practical problem of making a series of reticle widths thin enough to investigate the relative reticle-to-star-size problem.

This investigation showed that a microscope x-y stage capable of readout to 1×10^{-4} inch would be satisfactory and, because of the film scale factor, would yield 1 second of arc readout of the target position on the film. Although the limit of conventional mechanical micrometer readout is about 1×10^{-4} inch, higher precision can be obtained with high quality, fine pitch, lead screws, and nuts, such as those on astronomical comparators. These can be combined several ways (as indicated in ref. 7) to move the film or the microscope so that the target position may be read with a precision of 1 μ. Thus, the readout task can be accomplished with present technology.

Readout Reticle

An investigation was made to determine the effects of reticle pattern on the reading of the star image position on the film with the microscope. Figure 5 shows some of the reticles investigated with star targets. With reticle 1 the repetition of location of the center of the image of Polaris on Polaroid type 510 film had a standard deviation (σ) of 21×10^{-5} inch. For reticle 2, σ was 16×10^{-5} inch. As a result of centering measurements on one axis with various ratios of star-to-reticle width, it was concluded that a ratio of 1 was best. It became apparent that the rate of change of the visible lighted area, when the reticle was moved from the center, strongly influenced the ability to center accurately. A high rate of change of light flux received at the eye with reticle movement from the center leads to good centering position. Reticle 3 was designed with these criteria in mind. It incorporates good sensitivity characteristics, two-axis readout, and is adaptable to different sized star images. Computation of rate of change of exposed star area shows the rate to be highest when almost centered. The test showed a σ of 14×10^{-5} inch for repeatability of centering.

A reticle for locating a planet center might perform differently for certain conditions than one for locating a star. For example, the image of a star on high-speed Polaroid 510 film would be white on a dark background. A photograph of a planet taken on higher resolution Polaroid 55 negative film would give a dark planet image on a clear background. This reversal of target contrast prohibits the use of reticles 1, 2, and 3 for both targets. Reticle 4, however, could be adapted to both targets. Work reported in reference 8 indicates that a concentric circle reticle, such as reticle 4, may prove effective in locating a planet center. The standard deviations for centering the pseudo-planets (for the same but least proficient observer of that work) showed a σ of 22×10^{-5} inch for a full disk and 65×10^{-5} inch for a thick crescent. The σ values for reticle 4 correspond to a location precision between 1 and 6 seconds of arc for star and planet.

Field of View

The field of view depends on the choice of focal length and format size, and has important bearing on the utility of the photographic sextant. The use of a 24-inch focal length and the format size of 3-1/2 by 4-1/2 inch yields a field of view of 8.3° by 10.7°. The advantage of this relatively large field of view is that it eases the pointing requirements for target acquisition. For a typical lunar mission, the Earth and Moon images will not fill this field of view unless the spacecraft is less than 7 hours from injection or 5 hours from pericynthian. Many stars of a given constellation can also be acquired on the film to aid in the identification task. Figure 6 is a plot of all stars of 3.5 magnitude and brighter between latitudes of $\pm 80^\circ$. In the upper left corner, an inset indicates the format size to scale of the plot at 0° declination. In figure 6 there are several groups of stars near the ecliptic for which three or more stars would be within the field of view of the camera at one time. Stars dimmer than 3.5 magnitude will probably not be imaged satisfactorily in the short exposure times that will be used. Reference 8 indicates that stars can be identified within unknown sample areas of the celestial sphere for fields of view of 11° or more in less than 1 minute. The stars used in that study ranged to about sixth magnitude; they were probably identified by the distinctive patterns of the brighter stars. The dimmer background stars tended to obscure the distinctive patterns of the brighter stars. For this reason, and the additional fact that the astronaut will know the constellation at which he is aiming, bright star photographs should prove satisfactory for target star identification.

Film

High film speed for the star photographs is important to the photographic sextant for the following reasons:

(a) To allow short exposure time to avoid star streaks caused by vehicle or operator motion;

(b) To avoid image motion compensation to keep the instruments simple;

(c) To acquire sufficient stars, in the short exposure time, to easily identify the target star;

(d) To keep the aperture small and the instrument as small as possible.

The highest speed of film for the present tests was Polaroid 510, a high contrast film with ASA 10,000 speed rating. Low temperature and high humidity conditions prevailed at the time the test star photographs were taken. Despite precautions to protect the film, the environment did slow the film speed. Consistent speed results with Plux X film had been obtained in the laboratory previously, so speed tests of Polaroid films were compared to those for Plus X. The results are shown in figure 7.

If reflectance is defined as the ratio of reflected to incident light, transmittance as the ratio of transmitted to incident light, density as log(1/transmittance), and exposure as meter-candle-seconds, the plot of figure 7 can be seen to be similar to the usual density versus log exposure type curve for the films of this study. A 1/100-second exposure was used for all data points. Since Plus X is a negative film, the density is plotted as log(1/transmittance) while the counterpart for the Polaroid positives is plotted as (1/reflectance). The 0.1 density change starting at the toe of the characteristic curve of a negative, and at the shoulder for the positive prints, serves as the speed criterion and gives the minimum exposure for this film response. Point A, for example, is the speed index point for Kodak Plus X film. Thus, the American Standards Association tests PH 2-1966 and PH 2.5-1954, for paper and film, respectively, were not applied. The ASA speeds indicate a relative speed 6-1/4 and 4-1/2 stops faster than Plus X for Polaroid 510 and 57, respectively. The laboratory measurements (fig. 7) show that, on the basis of the 0.1 density change criterion, Polaroid 510 is 7 stops faster than Plus X, and further, that type 57 is 6-1/2 stops faster than Plus X and closely approaches the speed of type 510.

It must be possible to develop the film easily and quickly in the space environment. Dimensional stability of film emulsion and backing is important to retain targets in their true positions. For permanent astronomical star photographs, glass backing is traditionally used and developing is by wet process. The developing and drying operations are known to affect the spatial stability of the emulsion on photographic plates. Reference 9 reports on work directly applicable to the photographic sextant in this regard. A fine grid line pattern, 4 by 6 inches, on a transparent master was contact printed against photographic plates. The 48 intersection points of the master were compared with the same points on the prints after the prints were processed and spatial distortion was measured. The results indicate that the distortion to the spatial location of star images due to emulsion would cause an rms error less than 0.7×10^{-4} inch, which is less than 1 second of arc. Thus, if emulsions adapted to glass plates behave as described in reference 9, the star images would have sufficient spatial stability to meet the sextant requirements.

Image Fidelity

If the angle measurement of the photographic sextant is to be precise to a few seconds of arc, the star image must be located via reticle to at least the same precision. For the star image, some lack of fidelity can be tolerated if the degradation of the image is uniform about a center. However, since a planet landmark is not symmetrical, any degradation of the image would impair the ability to locate its position. Therefore, the highest possible fidelity for the landmark image would be demanded.

The combination of lens, film, and development process affects the fidelity of the image of the photographic sextant. For green mercury light, the Rayleigh criterion indicates a resolving power of 250 lines (pairs) per millimeter for the 24-inch EFL, 4-inch aperture optical system. In practice, reference 10 indicates that even more lines per millimeter can be resolved by a lens. Therefore, the need for a high resolution film for the planet camera is clearly indicated to keep the film compatible with the lens system.

Exposure Time - Image Motion Considerations

The exposure time is limited by the attitude limit cycle of the space vehicle in the navigation mode. The angular velocity of the vehicle will cause the stellar scene to appear to move past the window. For an Apollo lunar mission vehicle, it is estimated that there will be an angular velocity of 120 seconds of arc/second of time for a limit cycle with a small dead band. Thus, with a 1/25-second exposure, a star streak equivalent to 4.9 seconds of arc would result. The scale factor for the film makes this equivalent to 0.00057 inch. The image size of the 3.5 magnitude star was 0.02 inch. A 1/25-second exposure would result in an elongated star image 0.02 by 0.02057 inch. This slight eccentricity should not degrade materially the reticle centering precision.

Exposure Time - Photometric Considerations

The exposure time required from the standpoint of film sensitivity is a function of star image illuminance on the image plane, film speed, and flare factor. A sample calculation of this time is:

(1) Star image illuminance:

$$I_o = \frac{ID^2TH \cos^4 \theta}{d^2}$$

where

D aperture size, 4 in.

d size of star image, 0.02 in.

H vignetting factor, 0.9 (estimate)

I 9.67×10^{-8} lumen/m^2 for a 3.5 magnitude star

T transmission, 0.79 (estimate: window 0.9, lens 0.88)

θ maximum angle of off-axis ray for an 8.3° by 10.7° field of view, 6.775° half-diagonal

I_o 2.7×10^{-3} lumen/m^2

(2) Exposure time in seconds:

$$t = \frac{E_{min}}{I_o Q}$$

where

E_{min} $\frac{0.8}{Sx} = 8 \times 10^{-5}$

Q flare factor of lens, 1.7 (estimate)

Sx ASA film speed = 10,000

t $\frac{8 \times 10^{-5}}{2.7 \times 10^{-3} \times 1.7} = 0.018$ sec

This exposure is compatible with the exposure based on image motion considerations of 1/25 second.

In contrast to the star image, the Moon and Earth images will be so bright that the problem will be to limit light without decreasing the aperture to the point that diffraction limiting of resolution becomes a problem. Several avenues, such as shutter, filters, and film, are available for solving this problem, so it will not be examined further in this study.

Shutter

The shutter is a critical component for the sextant since vignetting must be avoided. Vignetting reduces the apparent magnitude of star images at the edge of the field relative to those close to the optical axis. One of the clues to identification of a target star is its magnitude relative to others in the photograph, thus, the design of the sextant must guard against vignetting.

Vignetting would occur if any rays from the 4-inch objective were cut off or obstructed before they could reach the 3-1/2- by 4-1/2-inch format. This means that a shutter between this lens and the format must provide a clear opening of at least 4 to 4-1/2 inches, depending on its location along the optical axis for a target on axis. A suitable large aperture shutter with a 1/100-second exposure time is not currently available. A focal plane shutter will not suffice because the complete field of view must be exposed to freeze the targets in their relative positions despite camera rotations. High tension, spring-driven, conventional shutters with large apertures have high starting accelerations that cause vibration during exposure. A venetian-blind-type shutter could be fast enough and yet not have objectionable vibrations but it would have to be placed close to the film to minimize diffraction effects.

There is current research on the use of various materials that exhibit electrooptical and electrochemical-optical effects. For their application as shutters, generally these materials have the disadvantage of low light transmission when open and incomplete light cut off when closed. It thus appears that research is needed to develop a large-aperture shutter suitable for the photographic sextant and its environment.

Window

The spacecraft window is, in effect, the first element of the optical system of the photographic sextant, and as such it has great influence on the optical system. If all errors that window imperfections might introduce are neglected, the angle change by refraction due to cabin pressure is illustrated in figure 8 for the two-line-of-sight sextant.

For targets in lines of sight at approximately equal incidence angles, the difference in incidence angles of the two lines of sight is expected to be less than 30° with the result that δ (angle between sextant angle bisector and window normal) becomes 15°. A measured change in α_M from 0° to 15° will cause a correction change in the sextant angle α of less than 1 second of arc.

An optimum window would have an infinite focal length lens assembly free of production anomalies, but in practice it will not be flat, the surfaces will not be parallel, the glass will not be homogeneous, and the deviation from flat will in all probability be regular enough that a radius of curvature will create a lens effect. These departures will cause image degradations as well as sextant angle error. The amount of such degradation and error in a Gemini left-hand (standard quality) window was measured and reported in reference 11. It appears that with the use of sufficient thickness-to-width ratio of panes, with proper frame design, and with specifications of glass and surface quality equivalent to Gemini right-hand window panes, errors in sextant measurements due to the window can be kept to less than 2 seconds of arc.

Angle Pickoff

The angle between the optical axes must be read precisely. Two choices are available: a smooth continuous change in angle which might employ an electrical analog, a digital pickoff, or a manual optical readout; secondly, discrete precision step changes in angle. The steps of, say, 5° could be made repeatable to 1 second of arc and calibrated in the laboratory. There would then be no requirement for optical or electronic readout with attendant instrument complexity. The photographic sextant is readily adaptable to this second choice because of the flexibility inherent in measuring star location anywhere within the field of view.

Any error due to misalinement between the optical axis and the mechanical readout axis of a sextant can be determined in the laboratory.

SCHEMATIC LAYOUT OF SEXTANT TYPES

Five specific optical configurations were considered in detail and are shown diagrammatically in figures 1 and 9 through 12. These configurations were chosen because they represent possible combinations of elements that have trade-off advantages for a photographic sextant. They are shown in each case behind a viewing window of a spacecraft. It is obvious in these figures how intimately the problems and arrangements of the sextant configuration are influenced by the window used. (In each case a 12-inch-diameter window with two 1-inch-thick panes 1/4 inch apart was considered.) The objective aperture used is 4 inches in diameter in each case, except in figure 9 which is a 6-inch aperture. The EFL in figure 9 is 12 inches, and, in the other cases, 24 inches; the film format size for each field of view is 3-1/2 by 4-1/2 inches.

The sextant of figure 9 photographs a planet and a star in the same field of view, and would be the type required by the procedure of reference 12 to obtain the necessary photographs of the Earth against its star background. Several optical and camera manufacturers have proposed and made somewhat similar cameras that could be modified for use as sextants.

The sextant of figure 10 is the usual marine sextant configuration except that the eye is replaced by film, and the targets need not be superimposed.

The sextant of figure 11 is like that of figure 10 except that the index mirror rotates about the reflected line of sight giving a 1:1 ratio of mirror rotation to sextant angle change unlike the usual sextant of figure 10, which gives a 1:2 ratio of mirror rotation to sextant angle change. The readout of the sextant of figure 11 is thus more sensitive and a 60° sextant angle change is possible.

Figure 12 incorporates most of the features of the preceding configurations and has several additional advantages. There is no overlapping of star and Moon images which might obscure the star. The two separate cameras allow use of two types of film: a slow-speed, high-resolution film for the Moon line of sight, and a high-speed film for the star line of sight. There is no loss of star light due to a beam splitter and a good 60° sextant angle can be obtained.

The sextant in figure 1 is practically identical to that in figure 12 except the former allows a larger 80° sextant angle. Figure 13 is a conceptual drawing of a hardware adaptation of figure 1. Sextant angle readout problems and hardware trade offs would influence the choice of the concept of figure 12 compared to that in figure 1.

FILM TEST RESULTS WITH PROTOTYPE SEXTANTS

Actual star and Moon photographs were obtained with a 24-inch focal length camera (fig. 14) which served to prove that the exposure time of 1/25 to 1/100 second demanded by the operational requirements could be used. Figure 15 is one such photograph of the belt stars of the constellation Orion. These stars of 2.05, 1.75, and 2.5 magnitude gave good images of about 0.02 inch in diameter on Polaroid 510 film when photographed in 1/20 second with a 4-inch aperture and 24-inch focal length lens system. The goal for the space sextant is the capture of a 3.5 magnitude star image outside the atmosphere under equivalent conditions. The loss in star light energy through a clear atmosphere is a function of angle from zenith of the star or its slant range through the atmosphere. From reference 13, the atmosphere attenuates a star at 30° from zenith (the angle of the photographs), about 1 magnitude. Thus, the capture of the 2.5 magnitude star image indicates the potential for capture of a 3.5 magnitude star image in space. The 1/20-second exposure was made under ambient conditions of very high humidity and low temperature (40° F). In the space cabin environment of low humidity and 70° F temperature, the film speed will be faster and hence the 1/25- to 1/100-second exposure time should be sufficient.

In the work with star images, false images caused by emulsion irregularities could be spotted with practice. The lack of symmetry of the false image and the sharp discontinuity from white to black of the false image rather than a fading out of white to black, as the diffraction pattern of a star would suggest, both give good indications of a false target.

The Moon photograph in figure 15 was taken on Polaroid 55 N film at 1/90-second exposure with a 2-inch aperture and 24-inch focal length camera. The Moon image is sharp, and the concentric circle centering reticle of reference 8 can be used to locate the center of the Moon image. Thus, the off-axis displacements of the Moon center from the optical axis can be measured directly from the negative.

SUMMARY OF RESULTS

Basic assumptions affecting all the factors of design and use were:

Package size	20 by 16 by 11 inches maximum
Film size	4 by 5 inches

These assumptions plus analysis led to the following:

Equivalent focal length	24 inches
Aperture	4 inches
Scale factor	1.16×10^{-4} inch
Field of view	8.3° by 10.7°

Tests conducted during the study led to the following:

Star image size	0.02-inch diameter
Readout microscope	30 power with 3-1/2- by 4-1/2-inch stage travel with precision to 1×10^{-4} inch or better
Readout reticle	Concentric circles plus cross hair
Film	Polaroid 57 emulsion on flat glass plates for stars, Polaroid 55 emulsion on flat glass plates for planets

The preceding factors plus analysis led to the following:

Exposure time	1/25 to 1/100 second
Shutter	4-inch aperture or larger needed. Studies to devise a satisfactory one are needed.
Schematic layout	Two-line-of-sight, two-camera sextant has distinct advantages

Reference to other work led to the following:

Fiducial lines	Paired lines exposed on the film plate at time of exposure to indicate optical axis and plane of sextant angle
Window	Errors due to glass can be made insignificant by design and specification for suitable quality. Cabin air refraction is not significant if large difference in line-of-sight angles is avoided.
Angle pickoff	Use of discrete precision precalibrated steps of about 5°

Of the factors affecting the design and use of a photographic sextant in space navigation, as studied in this work, only two, the film and the shutter, present problems not easily solved. In the case of film, improvements in speed, resolution, and development are being made by film manufacturers and others concerned with reproduction and copying. All such improvements will enhance the feasibility of the photographic sextant as they occur. The solution to the shutter problem should be simpler and more immediate; it should be possible to develop a satisfactory shutter based on present technology.

It is believed that all essential factors have been studied and that factors not included in this study will not compromise the feasibility of a photographic sextant.

The study shows that because of the arbitrarily assumed cabin space limitations, factors such as focal length, aperture, and format size are smaller than would otherwise be chosen. Because of this and of the high precision required, the optical system must be designed to avoid as much distortion and aberration as possible. This will require advanced iterative digital computer programs to tailor the design for this task. Expert designer control will be required to make trade offs to reduce the overall distortions and aberrations.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., 94035, Oct. 4, 1967
125-17-02-03-00-21

APPENDIX

COMPUTATION OF TRUE SEXTANT ANGLE α

A. STAR AND MOON ON SAME SIDE OF R LINE (FIG. 2)

Apply the law of cosines to spherical triangle SMH:

$$\cos MS = \cos SH \cos MH + \sin SH \sin MH \cos \gamma \tag{1}$$

Apply the law of cosines to the triangle MQH:

$$\cos MH = \cos MQ \cos QH + \sin MQ \sin QH \cos(90 - \beta) \tag{2}$$

since PH = MQ

$$\cos MH = \cos PH \cos QH + \sin PH \sin QH \sin \beta$$

since HQP is a right spherical triangle

$$\cos QH = \cos PH \cos PQ \tag{3}$$

Apply the law of sines to the same triangle:

$$\frac{\sin \beta}{\sin PH} = \frac{\sin 90^\circ}{\sin QH} \quad \text{or} \quad \sin \beta = \frac{\sin PH}{\sin QH} \tag{4}$$

Substitute equations (3) and (4) in equation (2):

$$\cos MH = \cos^2 PH \cos PQ + \sin^2 PH \tag{5}$$

Since $0 < MH < 90^\circ$

$$\sin MH = \sqrt{1 - \cos^2 MH} \tag{6}$$

Apply the law of sines to triangle HPU:

$$\frac{\sin \gamma}{\sin\left(90 - \frac{PQ}{2}\right)} = \frac{\sin 90^\circ}{\sin\left(90 - \frac{MH}{2}\right)} \tag{7}$$

$$\sin \gamma = \frac{\cos \frac{PQ}{2}}{\cos \frac{MH}{2}} \tag{8}$$

since $0^\circ < \gamma < 90^\circ$

$$\cos \gamma = \sqrt{1 - \left(\frac{\cos \frac{PQ}{2}}{\cos \frac{MH}{2}} \right)^2} \tag{9}$$

but

$$\cos^2 \frac{PQ}{2} = \frac{1 + \cos PQ}{2} \tag{10}$$

then

$$\cos \gamma = \sqrt{1 - \frac{1 + \cos PQ}{1 + \cos MH}} = \sqrt{\frac{-\cos PQ + \cos MH}{1 + \cos MH}} \tag{11}$$

Substitute equation (11) in equation (1):

$$\cos MS = \cos SH \cos MH + \sin SH \sin MH \sqrt{\frac{-\cos PQ + \cos MH}{1 + \cos MH}} \tag{12}$$

Substitute equation (6) in equation (12) and simplify:

$$\cos MS = \cos SH \cos MH + \sin SH \sqrt{(1 - \cos MH)(-\cos PQ + \cos MH)} \tag{13}$$

Finally, substitute equation (5) in equation (13):

$$\cos MS = \cos SH (\cos^2 PH \cos PQ + \sin^2 PH) + \sin SH \sqrt{[1-(\cos^2 PH \cos PQ+\sin^2 PH)][-\cos PQ+(\cos^2 PH \cos PQ+\sin^2 PH)]} \tag{14}$$

Simplify:

$$\cos MS = \cos PH \cos(SH + PH) \cos PQ + \sin PH \sin(SH + PH) \tag{15}$$

$$\cos \alpha = \cos Y_M \cos(Y_S) \cos(\alpha_M - X_M + X_S) + \sin Y_M \sin(Y_S) \tag{16}$$

where X and Y follow the signs of conventional orthogonal coordinate systems centered on the lines of sight.

B. STAR AND MOON ON OPPOSITE SIDES OF R LINE (FIG. 3)

From right spherical triangle MQP

$$\cos MP = \cos QM \cos QP \qquad (1)$$

Use the law of cosines for the spherical triangle MPS

$$\cos MS = \cos MP \cos SP + \sin MP \sin SP \cos(90 + \delta) \qquad (2)$$

where δ is the angle QPM.

Substitute equation (1) in equation (2) and simplify:

$$\cos MS = \cos QM \cos QP \cos SP - \sin MP \sin SP \sin \delta \qquad (3)$$

By the law of sines,

$$\frac{\sin QM}{\sin \delta} = \frac{\sin MP}{\sin 90^\circ} \quad \text{and} \quad \sin MP = \frac{\sin QM}{\sin \delta} \qquad (4)$$

Substitute equation (4) in equation (3)

$$\cos MS = \cos QM \cos QP \cos SP - \sin QM \sin SP \qquad (5)$$

$$\cos \alpha = \cos(\alpha - X_M + X_S)\cos(-Y_M)\cos Y_S - \sin(-Y_M)\sin Y_S \qquad (6)$$

where X and Y follow the signs of conventional orthogonal coordinate systems centered on the lines of sight.

REFERENCES

1. Smith, Gerald L.: Secondary Errors and Off-Design Conditions in Optimal Estimation of Space Vehicle Trajectories. NASA TN D-2129, 1964.

2. Smith, Gerald L.; and Harper, Eleanor V.: Midcourse Guidance Using Radar Tracking and On-Board Observation Data. NASA TN D-2238, 1964.

3. McLean, John D.; Schmidt, Stanley F.; and McGee, Leonard A.: Optimal Filtering and Linear Prediction Applied to a Midcourse Navigation System for the Circumlunar Mission. NASA TN D-1208, 1962.

4. Mayo, Alton P.; Hamer, Harold A.; and Hannah, Margery E.: Equations for Determining Vehicle Position in Earth-Moon Space From Simultaneous Onboard Optical Measurements. NASA TN D-1604, 1963.

5. Blackwell, H. Richard: Contrast Thresholds of the Human Eye. J. Opt. Soc. Am., vol. 36, Nov. 1946, pp. 624-643.

6. Kissam, Phillip: Optical Tooling for Precise Manufacture and Alignment. McGraw Hill Book Co., Inc., 1962.

7. Whitehead, T. N.: Instruments and Accurate Mechanism. Dover Pub., 1954.

8. Hegarty, Daniel M.; and Thigpen, Marvin H.: Two Sighting Problems Associated With Sextant Type Measurements for Space Navigation: The Effect of Field of View on Star Field Identification. Preliminary Measurements of the Accuracy of Manual Optical Sighting at the Center of a Planetary Disk. NASA TN D-1653, 1963.

9. Altman, J. H.; and Ball, R. C.: On the Spatial Stability of Photographic Plates. Phot. Sci. Eng., vol. 5, no. 5, Sept.-Oct. 1961, pp. 278-282.

10. Altman, J. H.: A Simple Camera for the Measurement of Photographic Resolving Power. Phot. Sci. Eng., vol. 5, no. 1, Jan.-Feb. 1961, pp. 17-20.

11. Walsh, Thomas M.; Warner, David N., Jr.; and Davis, Michael B.: The Effects of a Gemini Left-Hand Window on Experiments Requiring Accuracy in Sighting or Resolution. NASA TN D-3669, 1966.

12. Havill, C. Dewey: An Emergency Midcourse Navigation Procedure for a Space Vehicle Returning From the Moon. NASA TN D-1765, 1963.

13. Hardie, Robert H.: Photoelectric Reductions. Ch. 8 of Astronomical Techniques, W. H. Hiltner, ed., Univ. of Chicago Press, 1962, pp. 170-208.

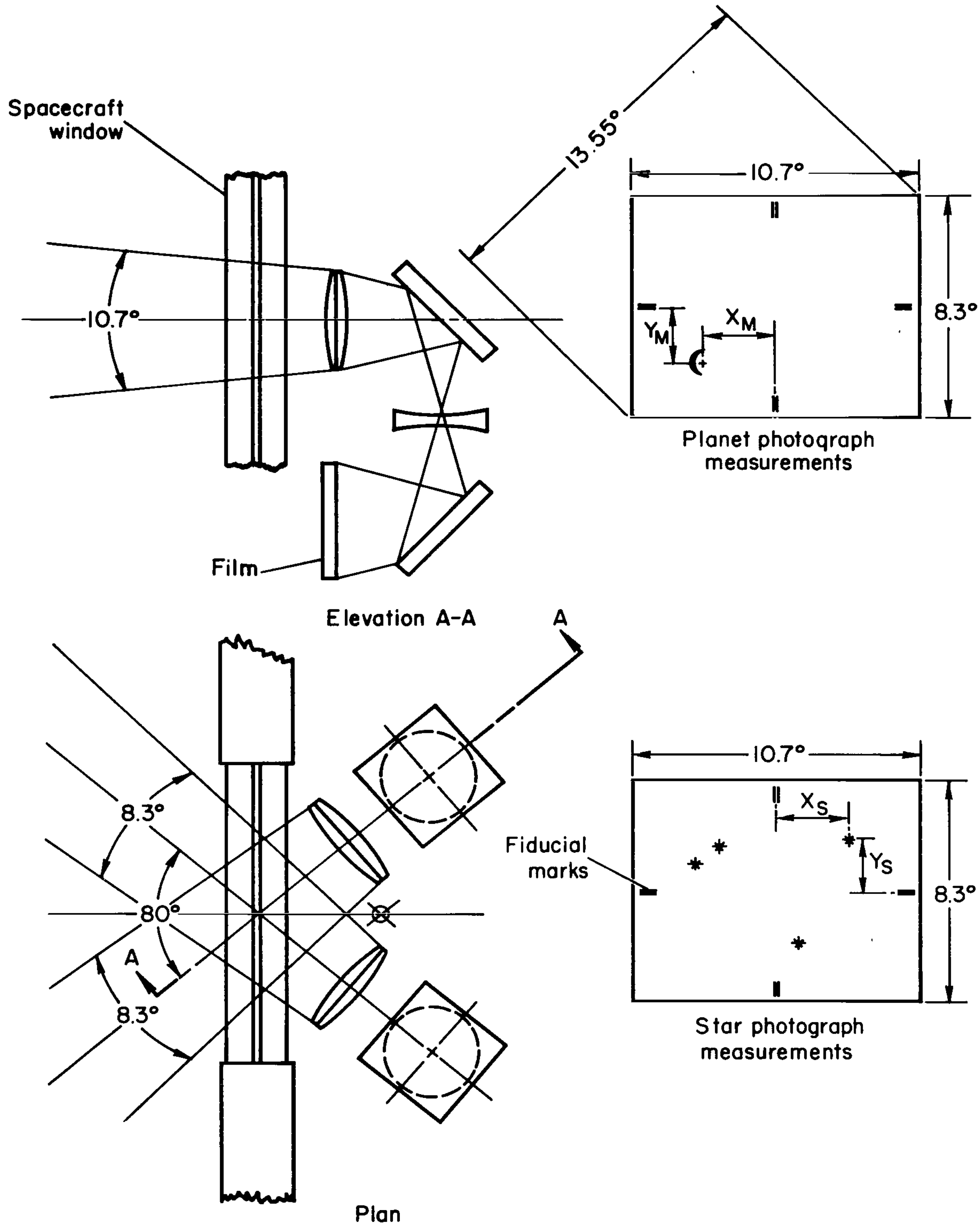


Figure 1.- Schematic diagram and types of measurements for a two-line-of-sight, two-camera sextant.

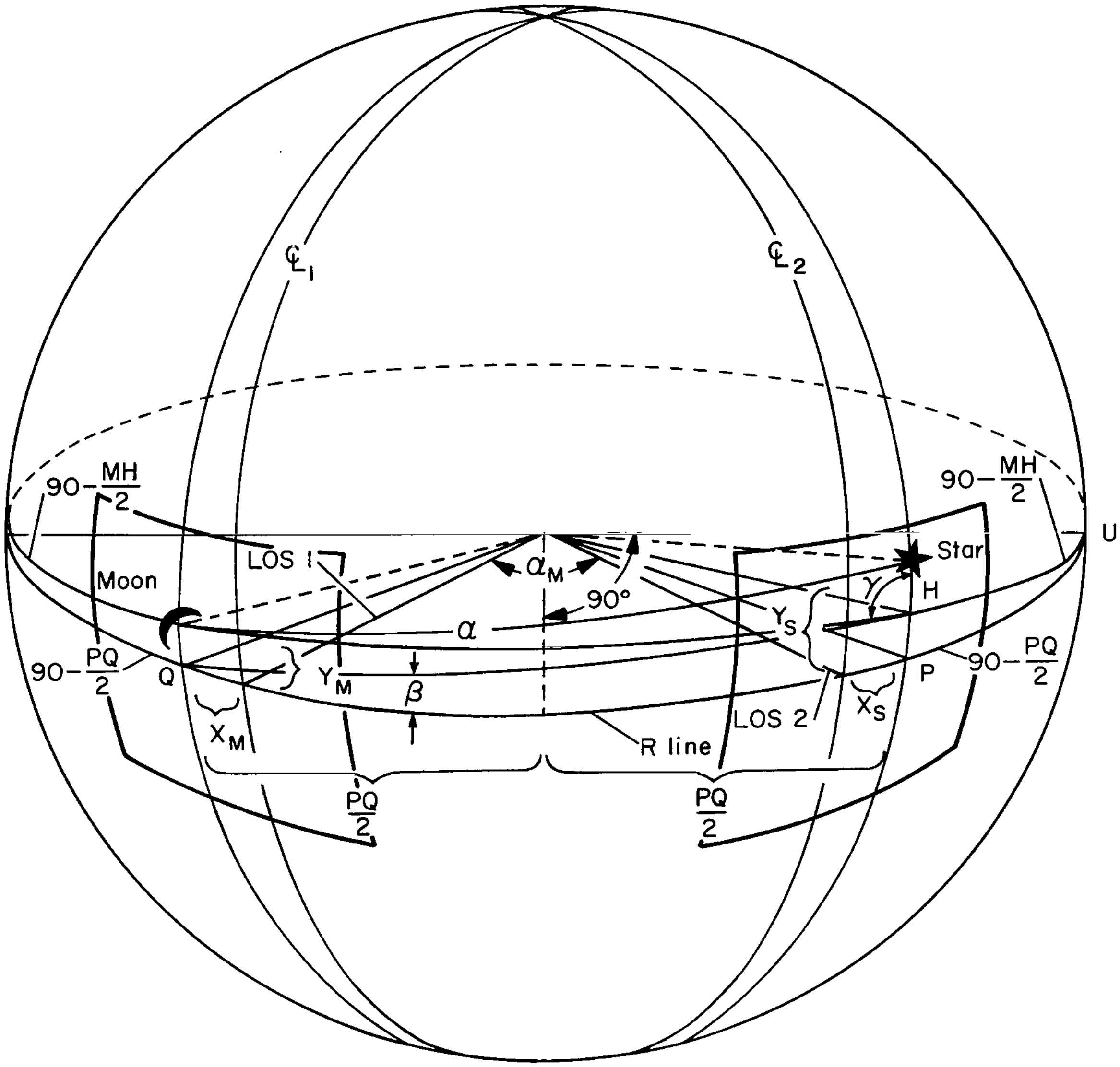


Figure 2.- Computation of true sextant angle α; star and moon on same side of R line.

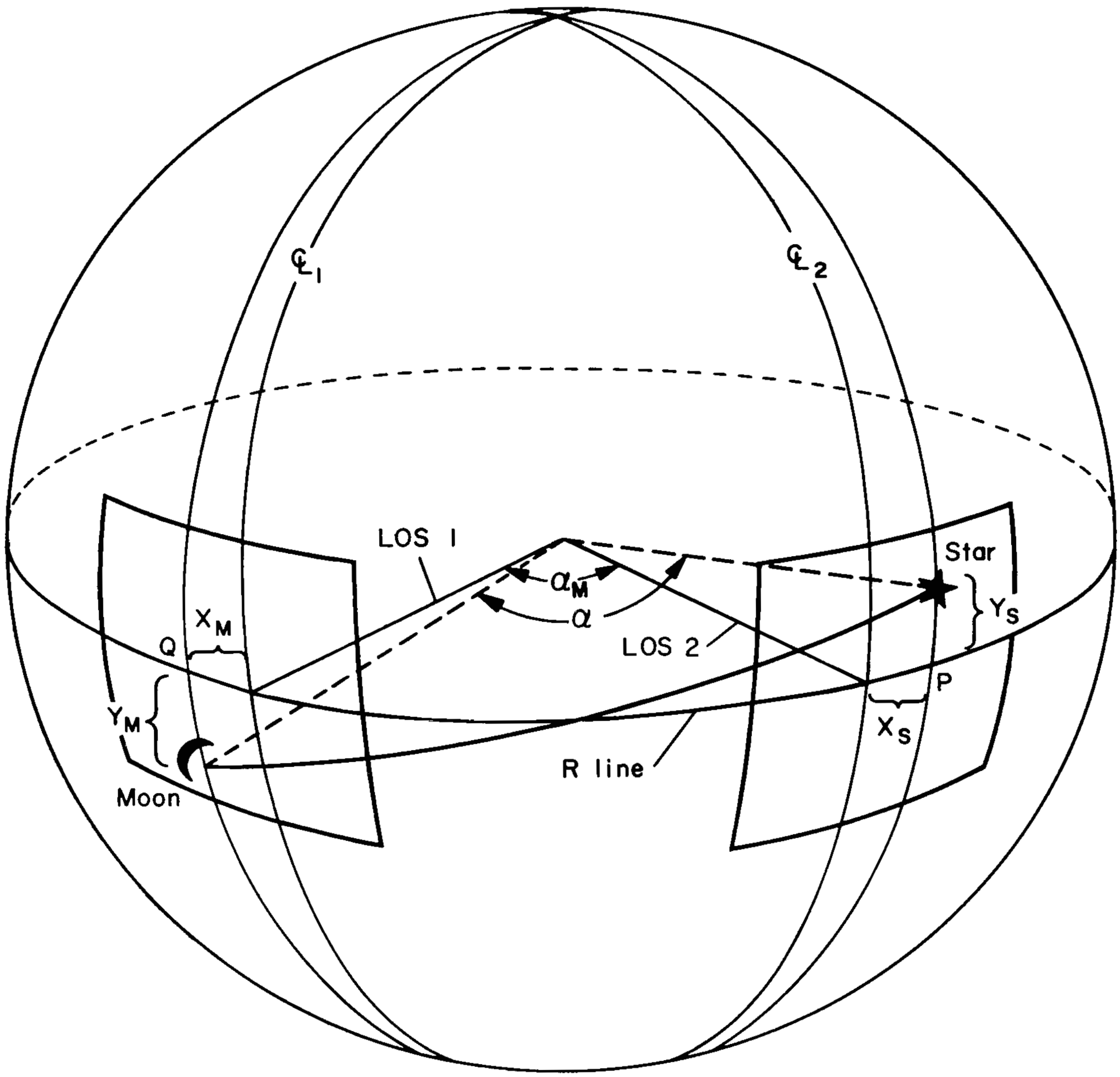


Figure 3.- Computation of true sextant angle α; star and moon on opposite sides of R line.

Aperture	f/number when f/number of negative lens is a maximum			
	EFL = 24"		EFL = 20.6"	
	Positive lens (a)	Negative lens (b)	Positive lens (a)	Negative lens (b)
4"	3	−2	3.25	−3.5
5"	2.4	−1.6	2.6	−2.8
6"	2	−1.33	2.17	−2.33

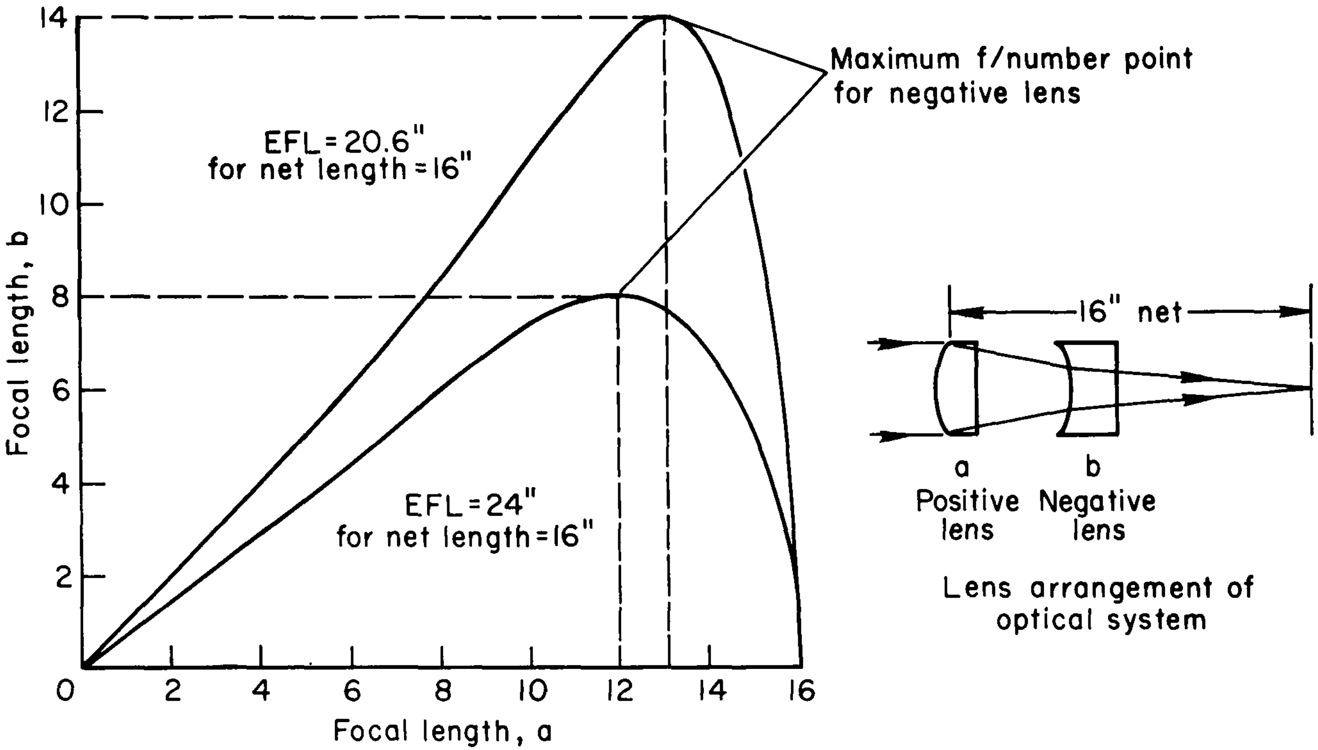


Figure 4.- Focal length of positive and negative lenses for net length of 16 inches for two different effective focal lengths.

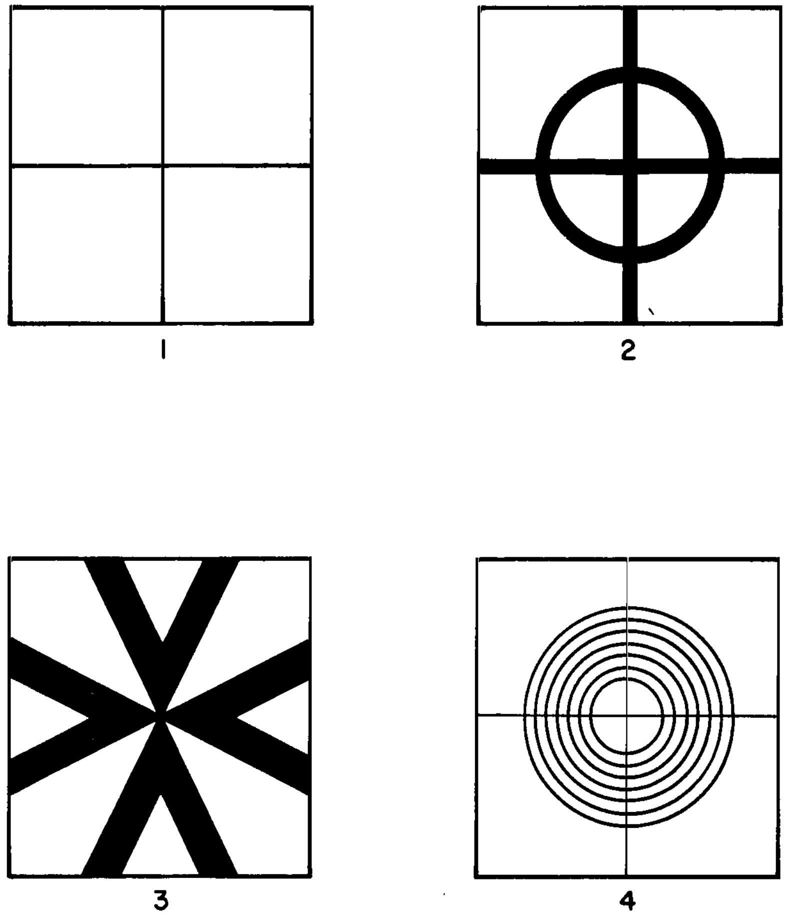


Figure 5.- Microscope reticles investigated.

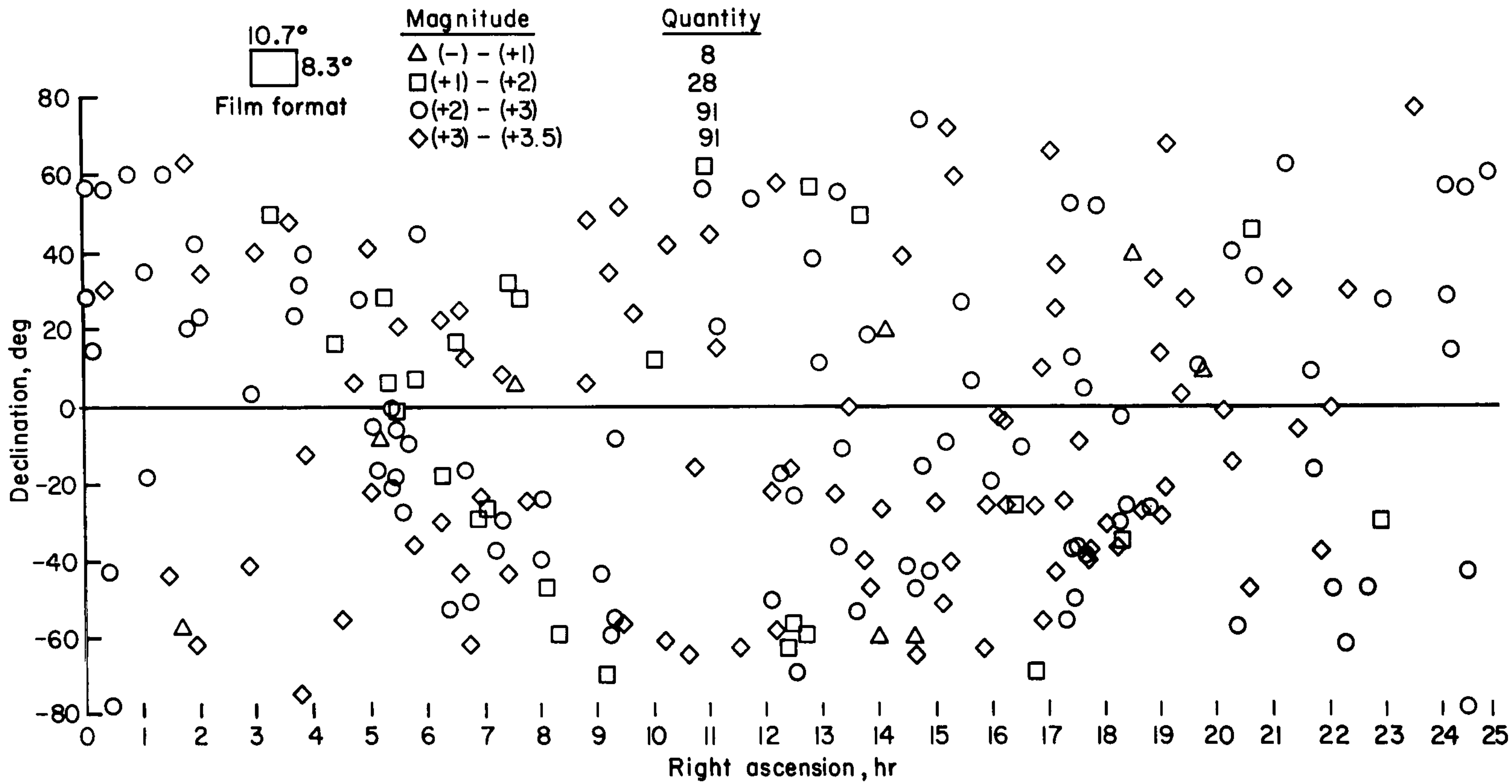


Figure 6.- Distribution of 3.5 magnitude or brighter stars.

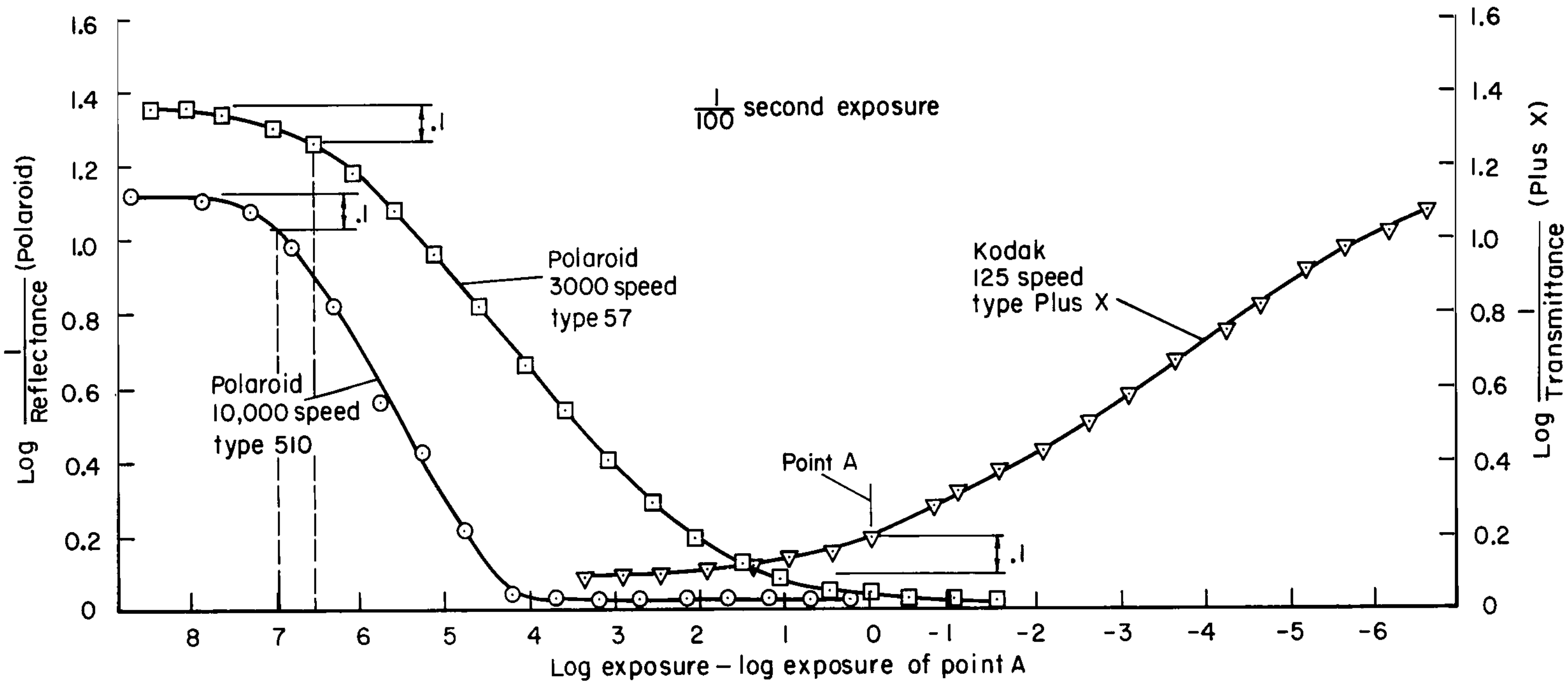
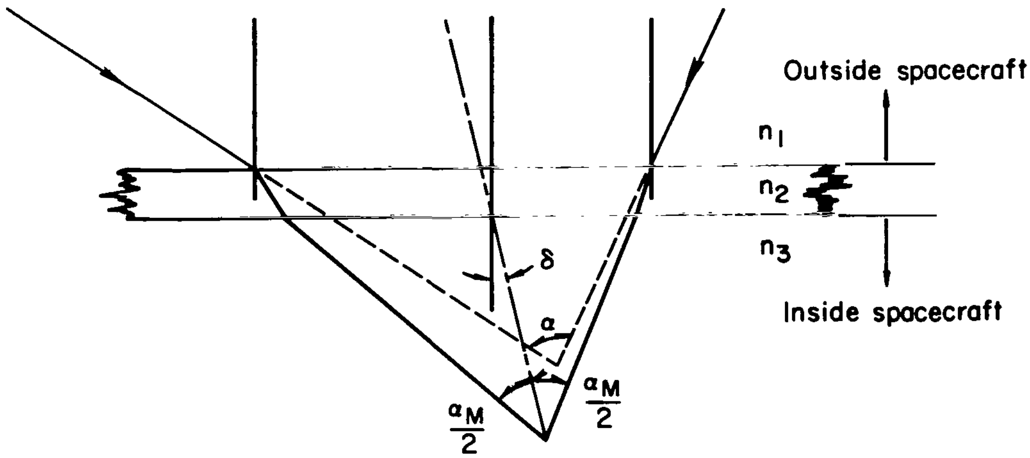


Figure 7.- Relative film speed.



$$\phi = \alpha - \alpha_M = \sin^{-1}\left[n_3 \sin\left(\frac{\alpha_M}{2} + \delta\right)\right] + \sin^{-1}\left[n_3 \sin\left(\frac{\alpha_M}{2} - \delta\right)\right] - \alpha_M$$

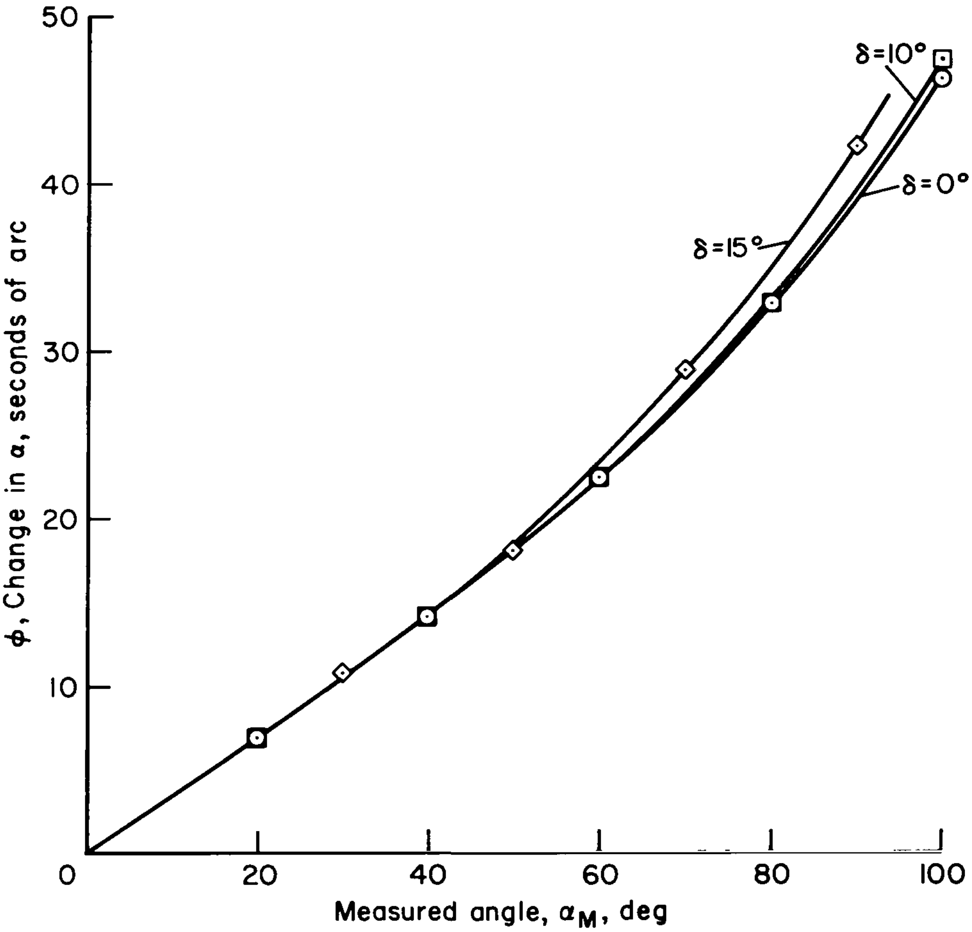


Figure 8.- Angle change by refraction of cabin air.

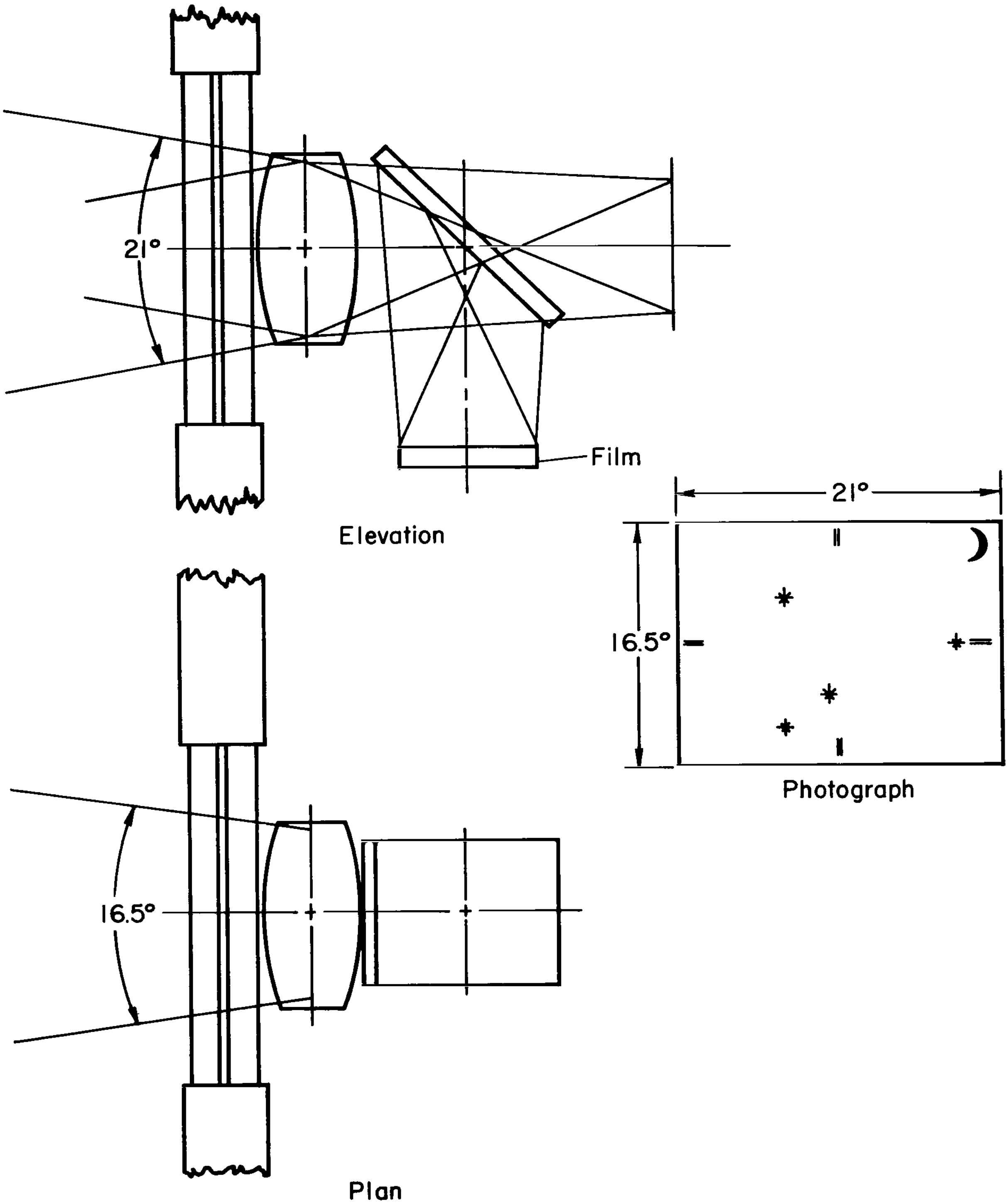


Figure 9.- Schematic diagram and types of measurements for a single-line-of-sight, single-camera sextant.

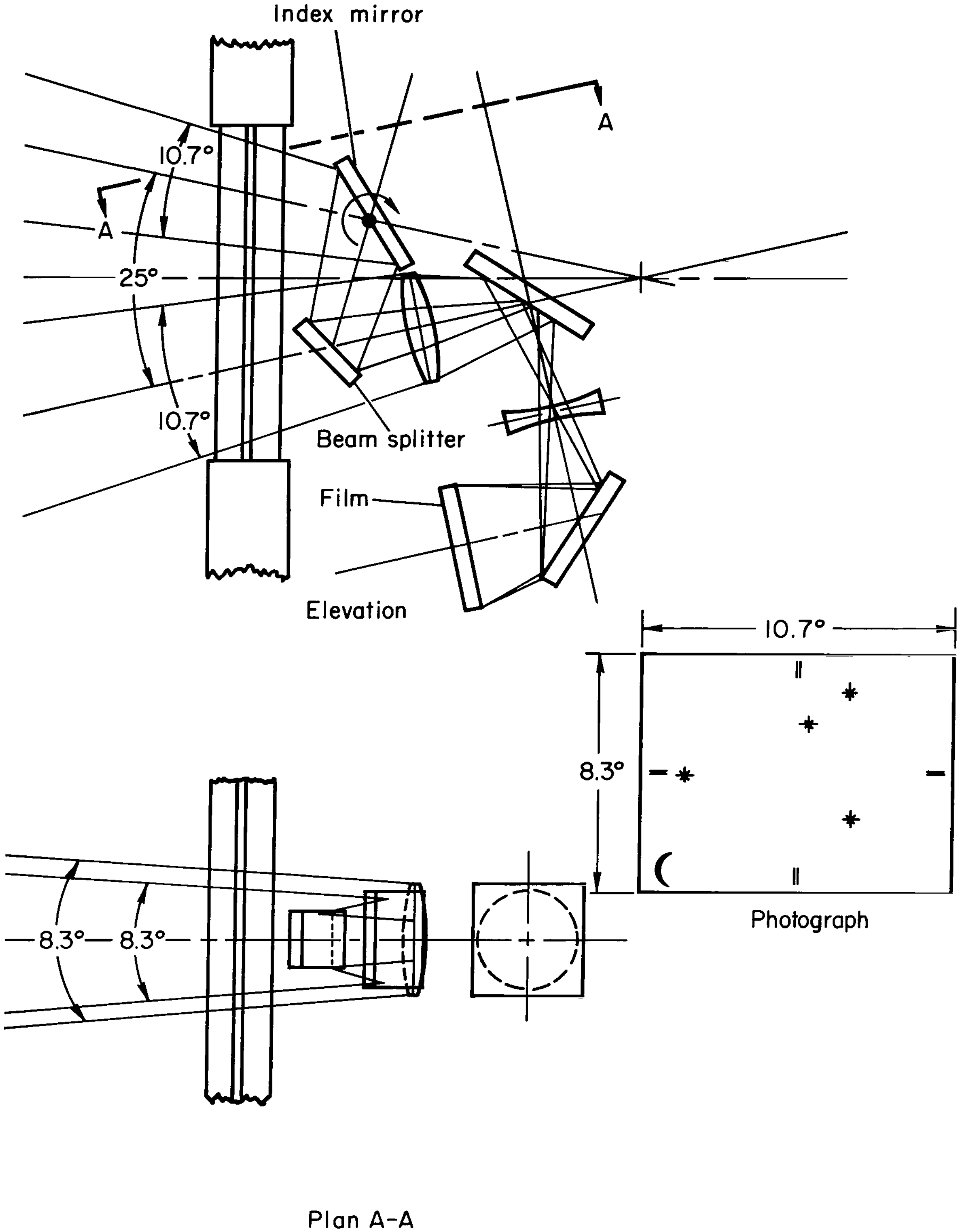


Figure 10.- Schematic diagram and types of measurements for a two-line-of-sight, single-camera sextant.

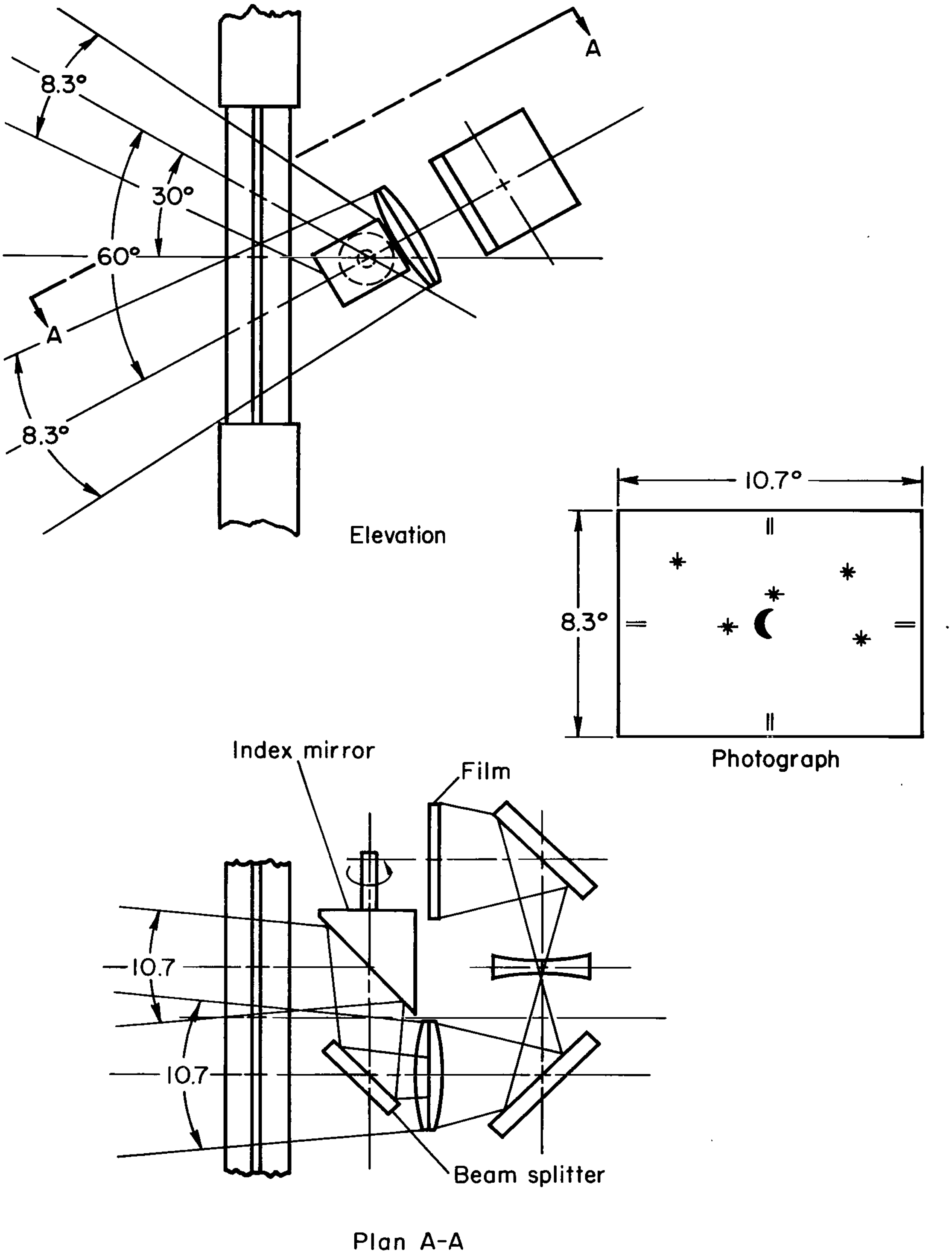


Figure 11.- Schematic diagram and types of measurements for a two-line-of-sight, single-camera sextant.

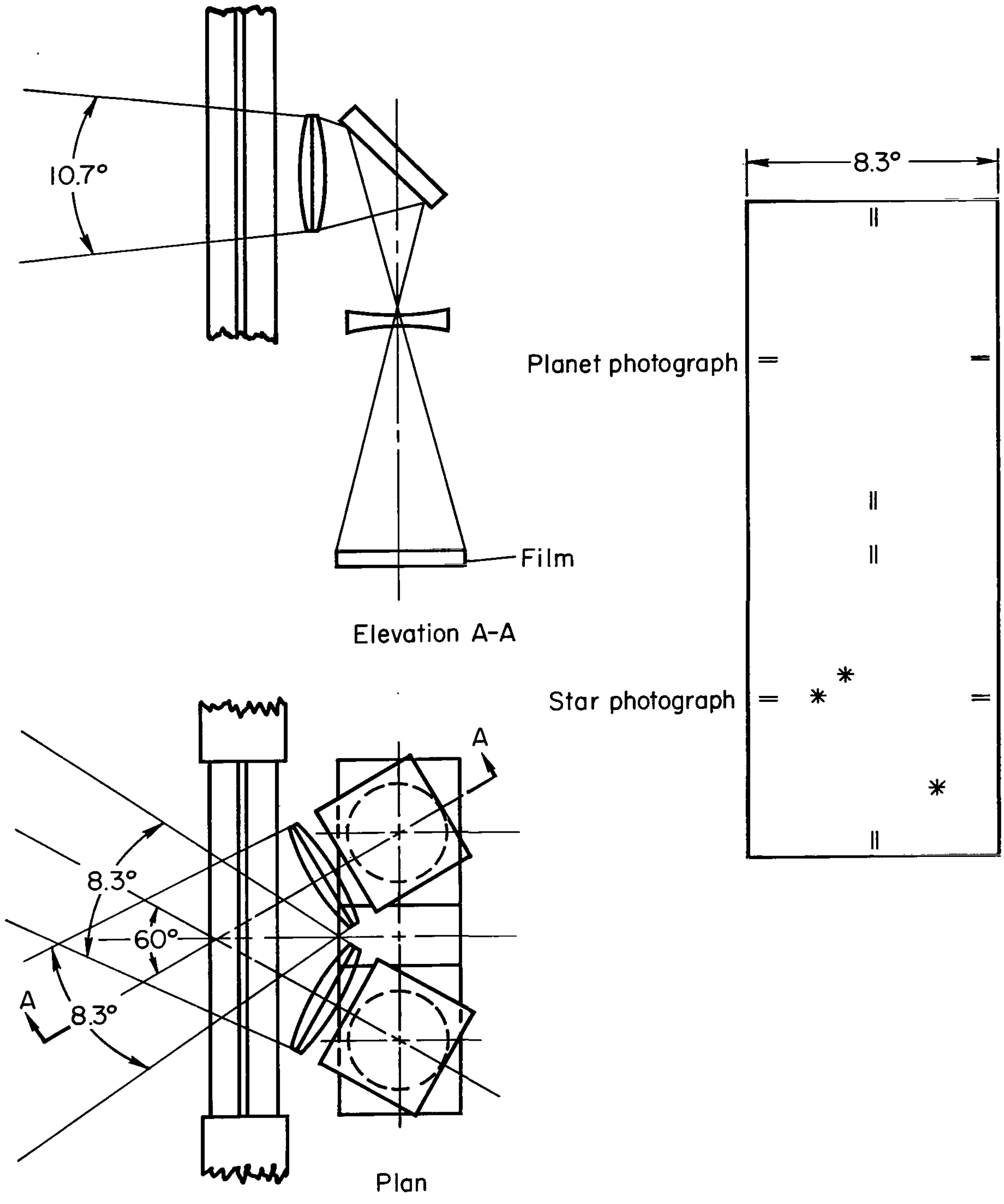


Figure 12.- Schematic diagram and types of measurements for a two-line-of-sight, double-camera sextant.

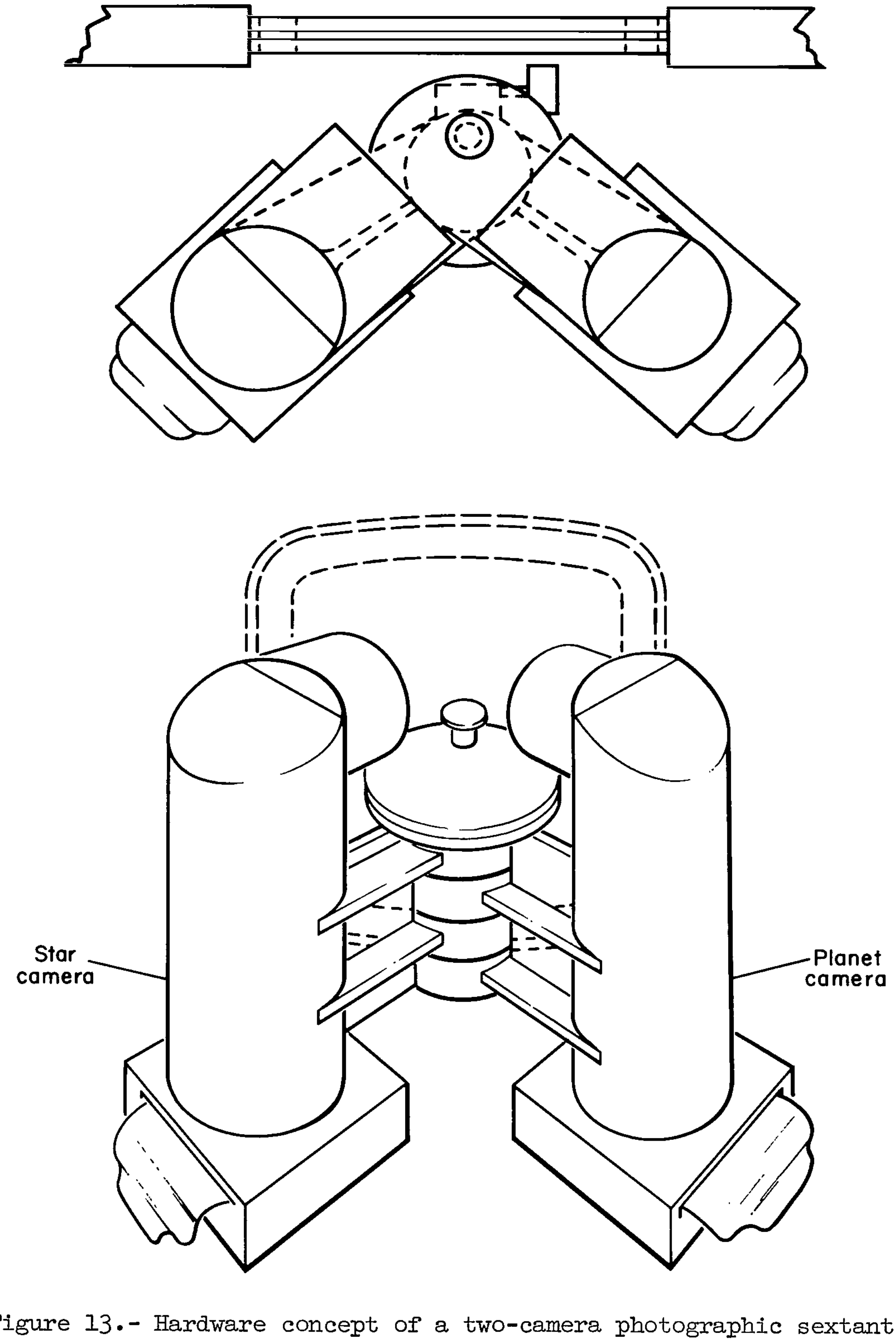


Figure 13.- Hardware concept of a two-camera photographic sextant.

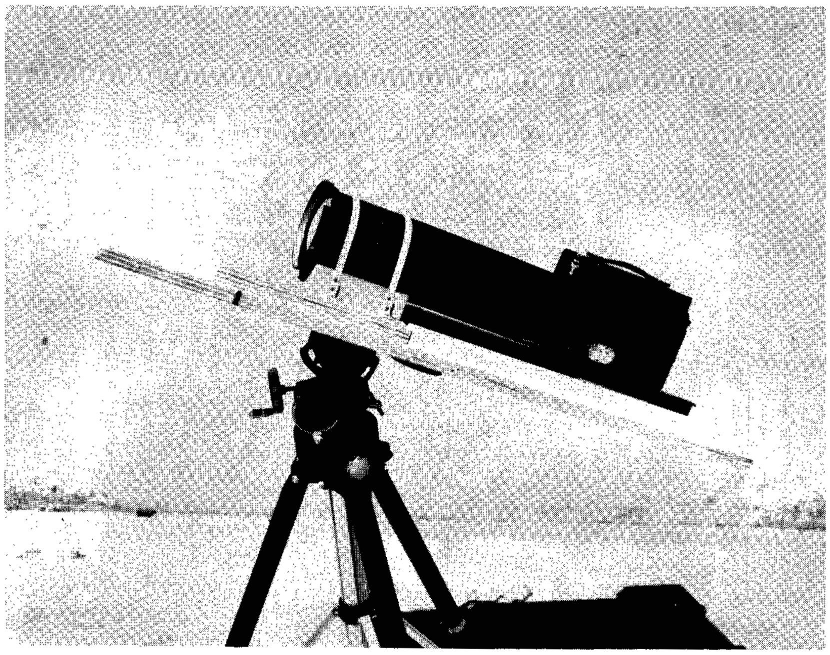
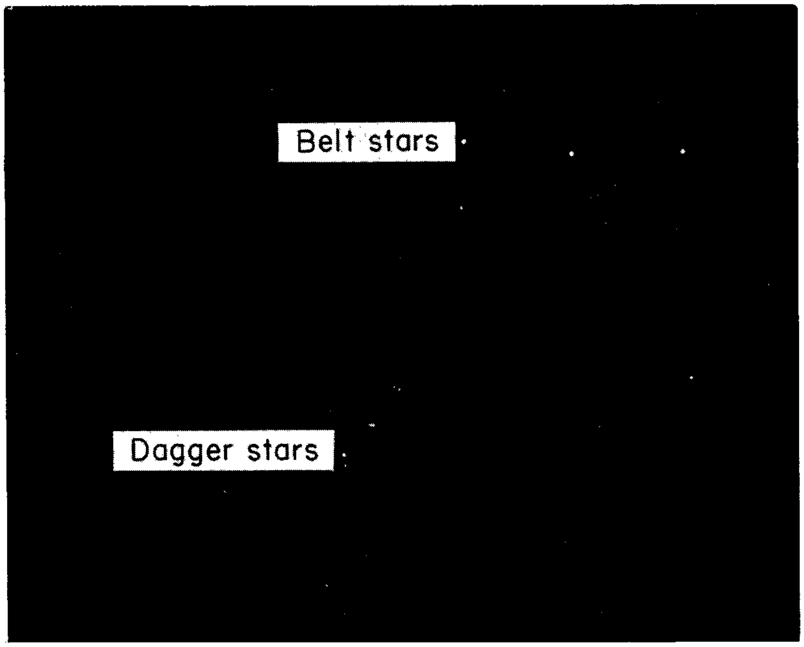
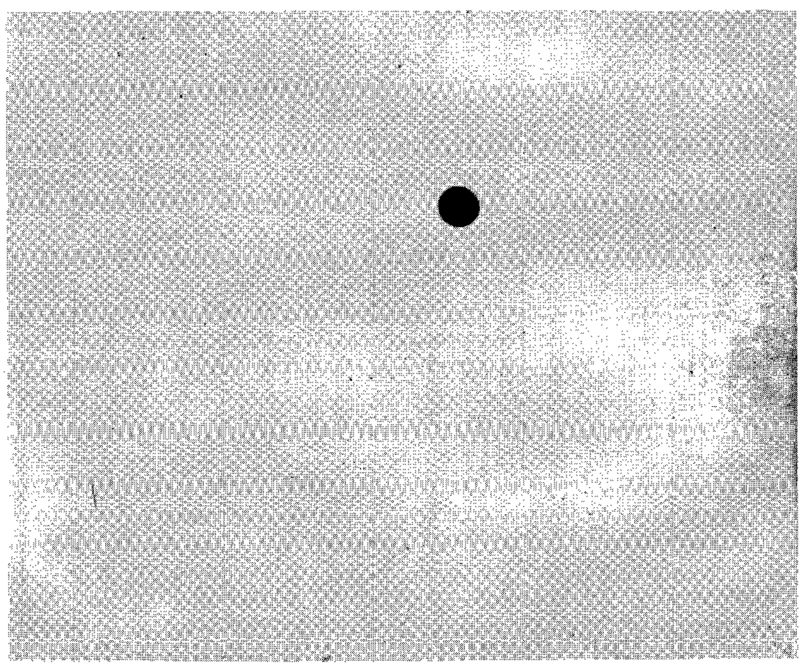


Figure 14.- 24-inch focal length camera.



(a) Star photograph.



(b) Moon photograph.

Figure 15.- Target images.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546